Quantitative DEM of dense granular packings with a multiple contacts force model

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Motivation

1. 3D Experiments

Access to the micro-structure, 3D force vectors

Used as a reference for a comparison with DEM

2. DEM

Usual DEM + Hertz interactions not good enough

Interactions between multiple contacts

Accessing the micro-structure

Mukhopadhyay *et al.*
Phys. Rev. E 84, 011302, 2011
Dijksman *et al.*
Rev. Sci. Intstrum. 2012

X-rays / micro-ct

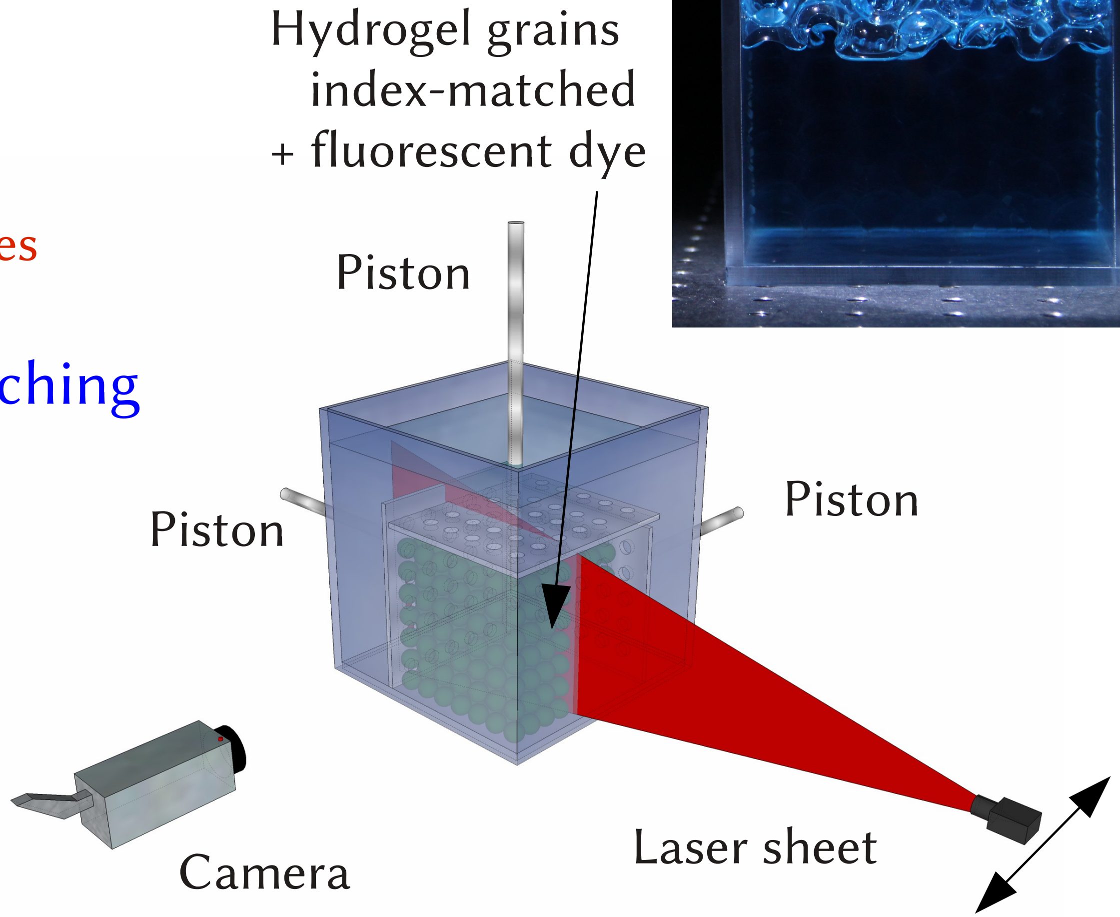
- Fine resolution
- Most materials
- Costly

Confocal: emulsions

- Microscopic
- Costly
- Difficult to control applied stresses

This work: refractive index matching

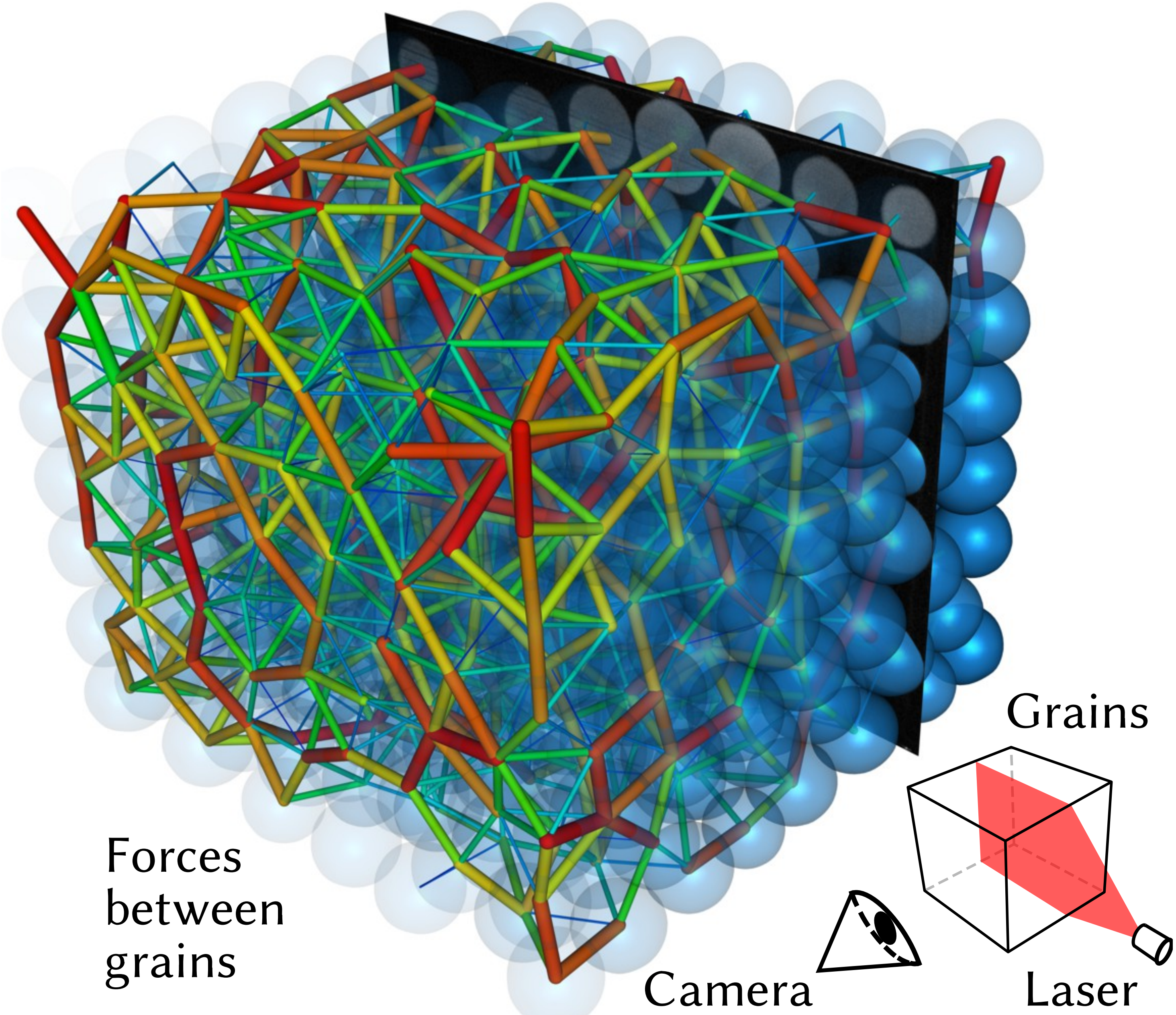
- Macroscopic grains
- Easy to control, tri-axial shearing
- Cheap
- Submersed



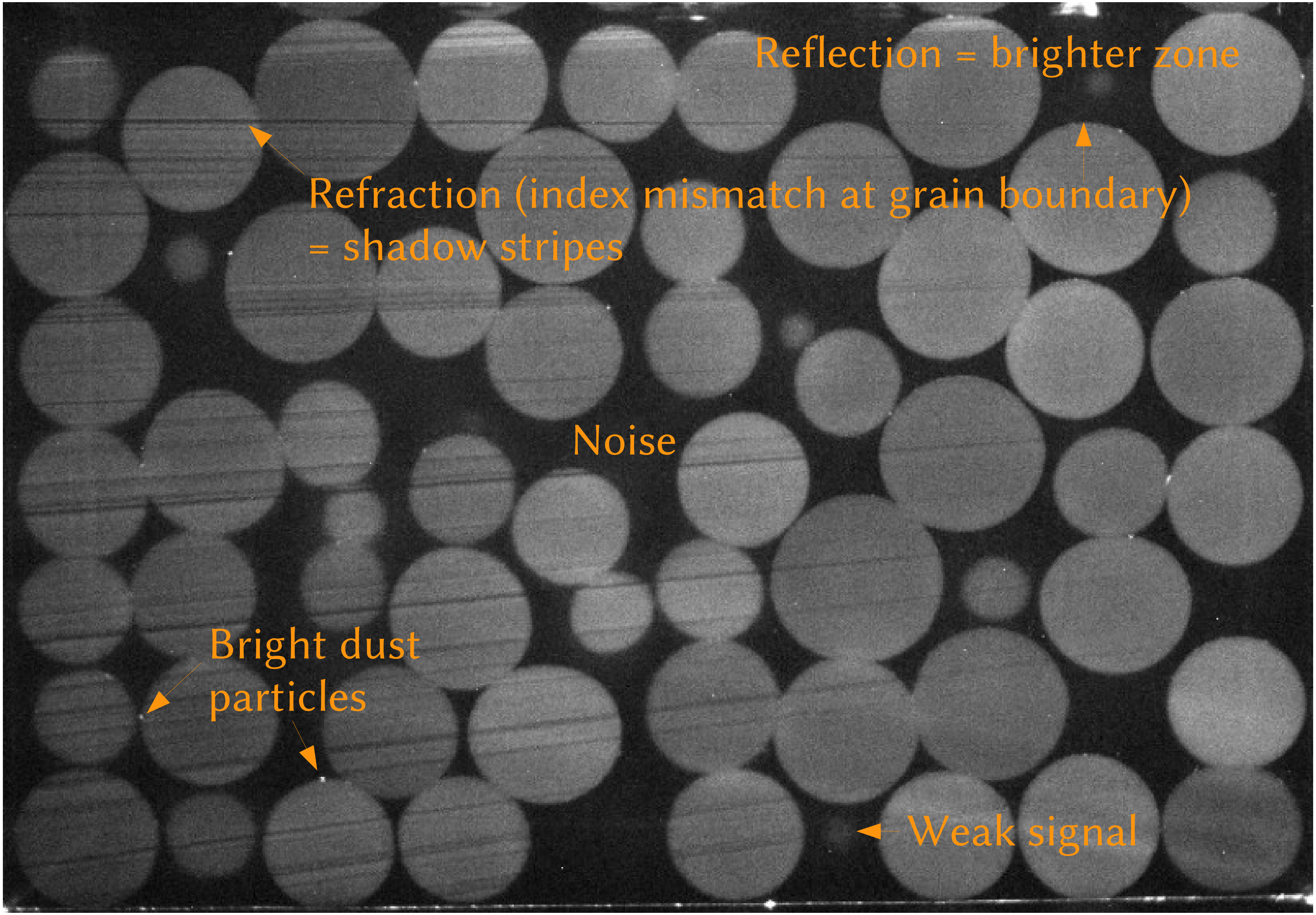
Next slides on:

1. Structure
2. Forces in 3D

What we get

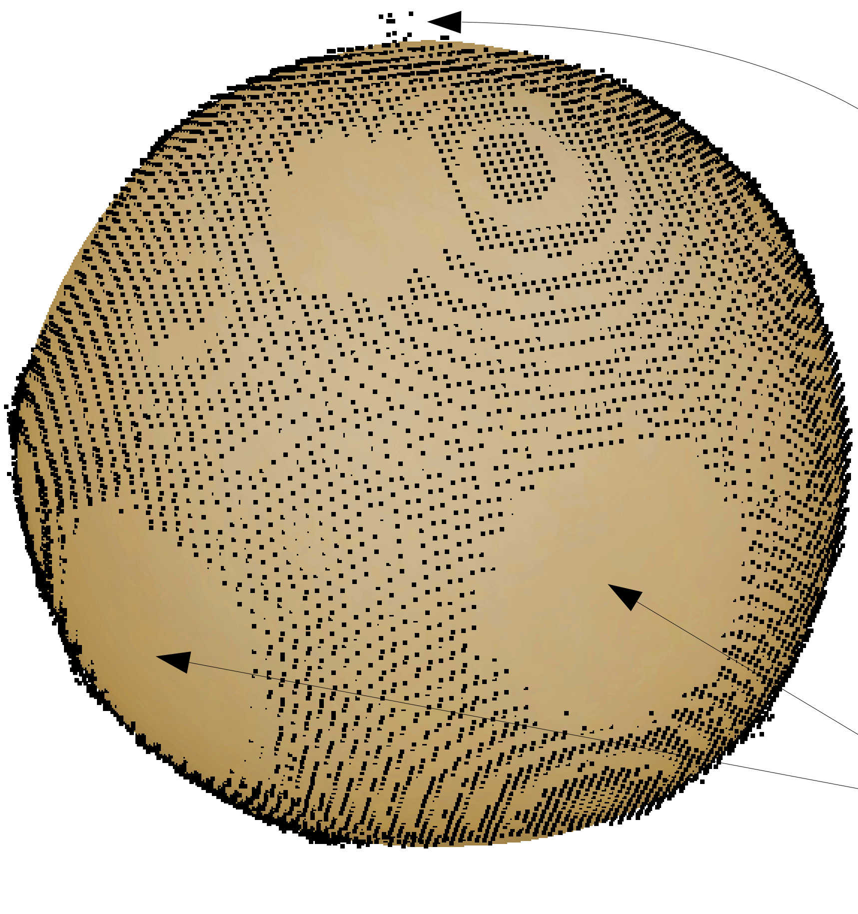


Typical image



From 2D images to 3D grains

Step 1: Stack the images into 3D voxels
Step 2: Detect border voxels and assign them to unique grains
Step 3: Fit an analytic surface to these borders
Step 4: Use these surfaces to get accurate forces

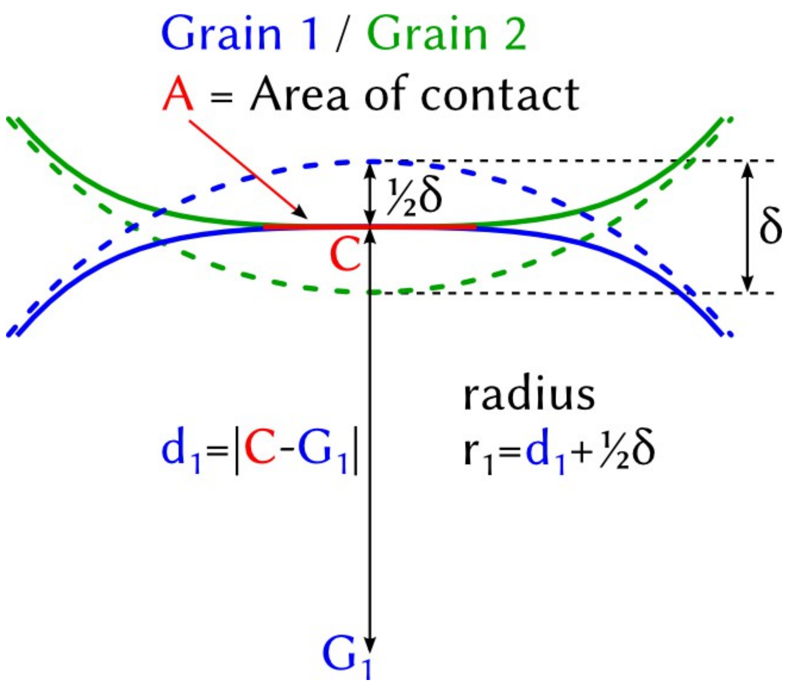


Outliers completely eliminated

Contacts = no border between grains
BUT surface area is well defined

Inferring forces in full 3D

Analytic shape descriptions $\Longrightarrow$ contact properties



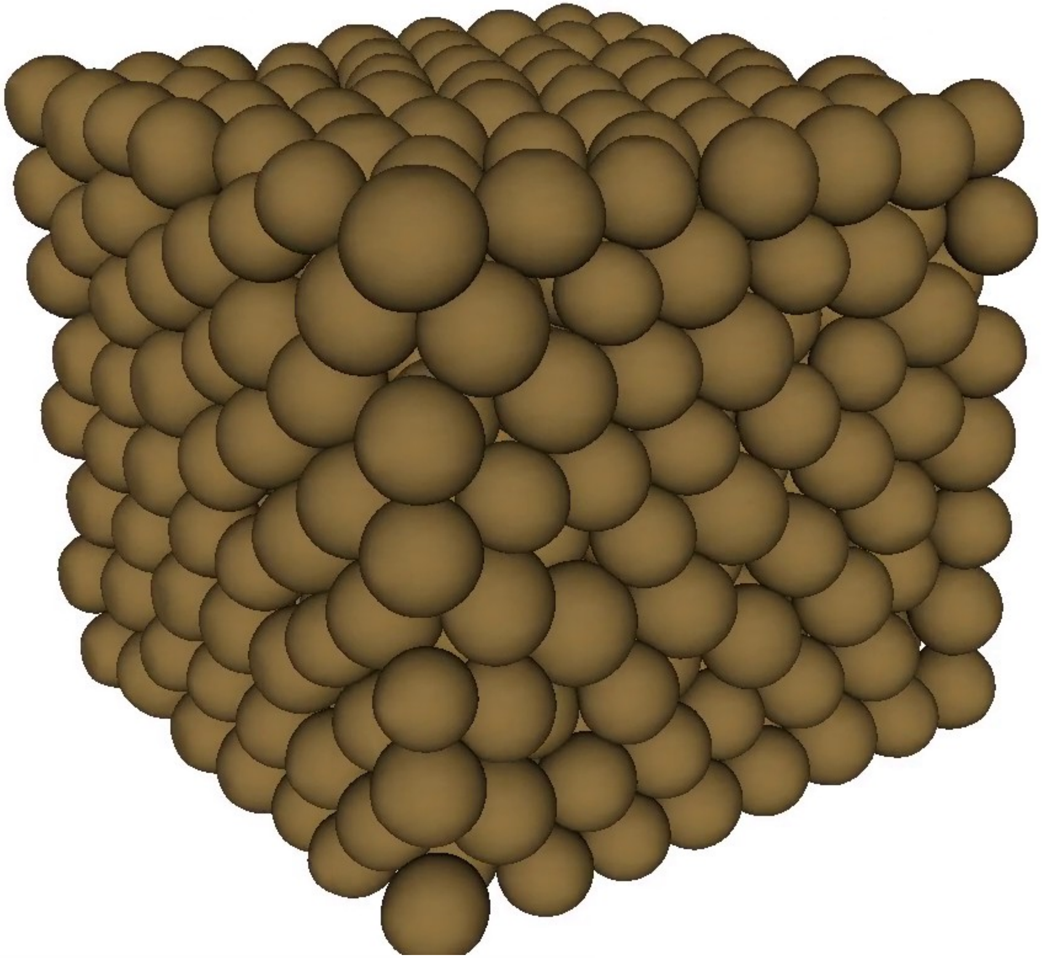
Measured here: A, C, G_1, G_2, d_1, d_2.
Unknown: δ

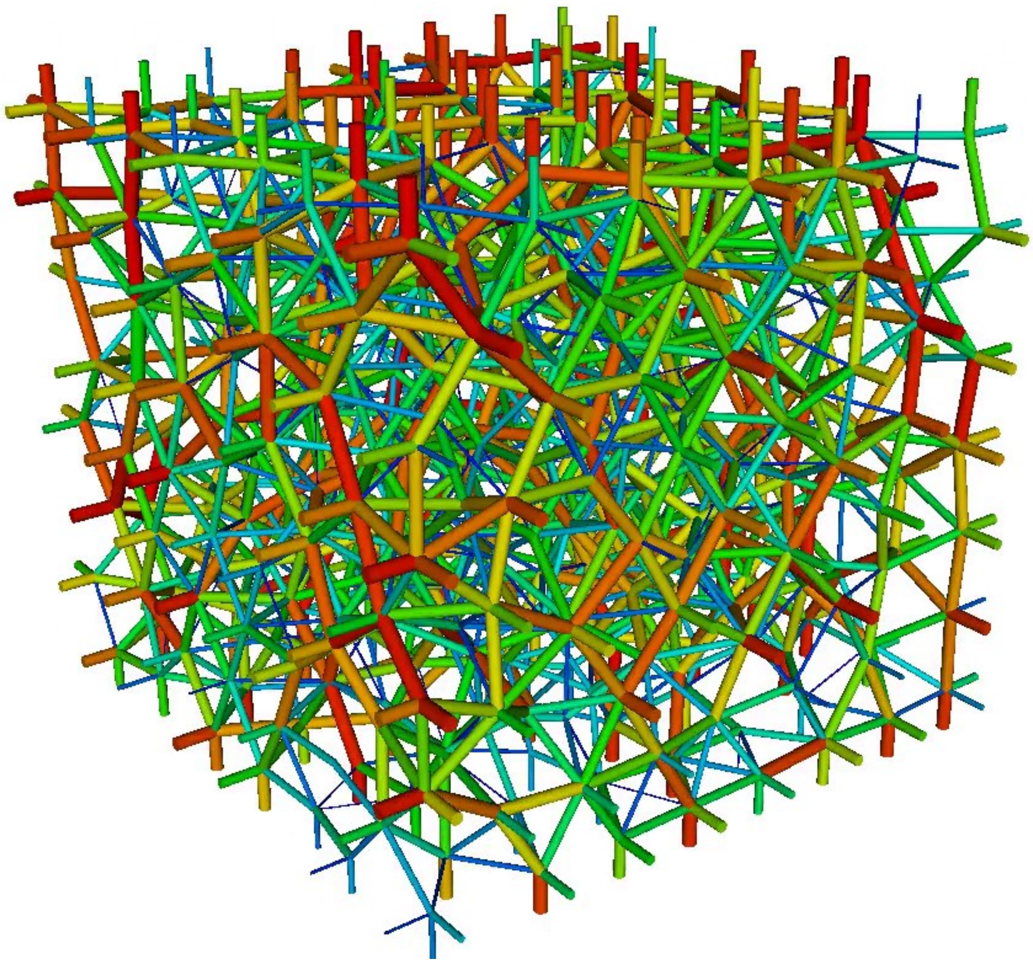
Contact properties $\Longrightarrow$ forces

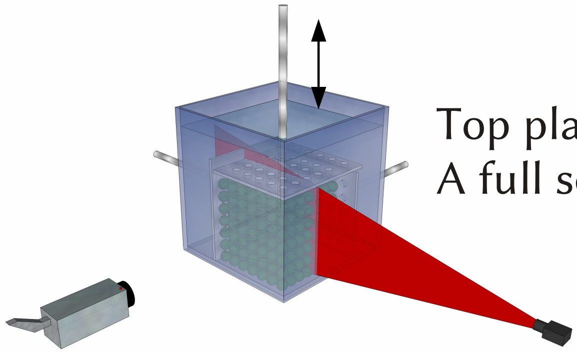
- $1/r = 1/r_1 + 1/r_2$ radius of curvature at contact
- $F = E\, r^{1/2}\, \delta^{3/2}$ E = effective Young modulus
- $F = E\, \delta\, a$ $a = \sqrt{A/\pi}$ radius of the contact
- Hence $r\,\delta = A/\pi \Longrightarrow \delta \Longrightarrow F$ (with given E)

$\Longrightarrow$ Vector forces in full 3D, with orientation, position, norm
+ grain centers of mass, stress tensor, etc.

Uni-axial compression cycles



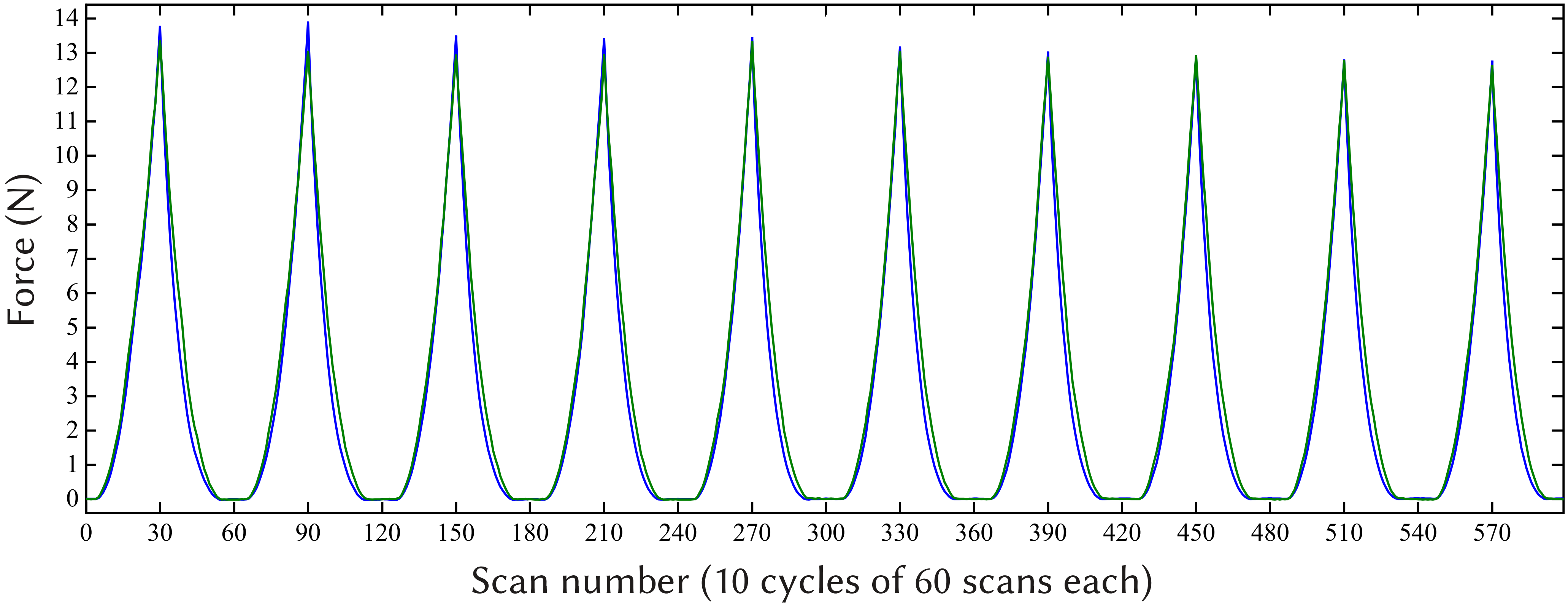




Top plate moves by 1mm increments
A full scan is taken between increments

Forces = struts joining the grain centers
Blue = weakest, Red = strongest

Validation on compression cycles

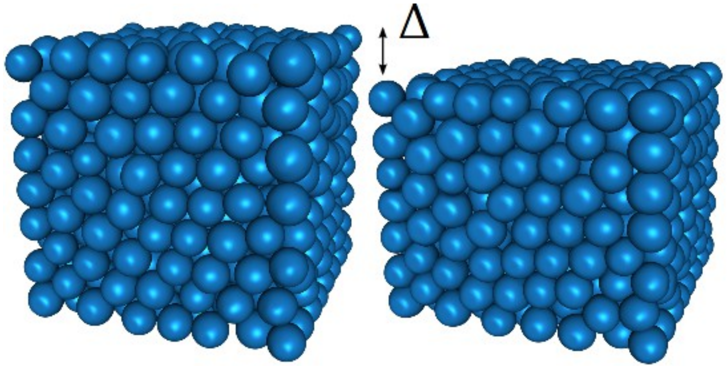


Blue = force measured on the top plate sensor

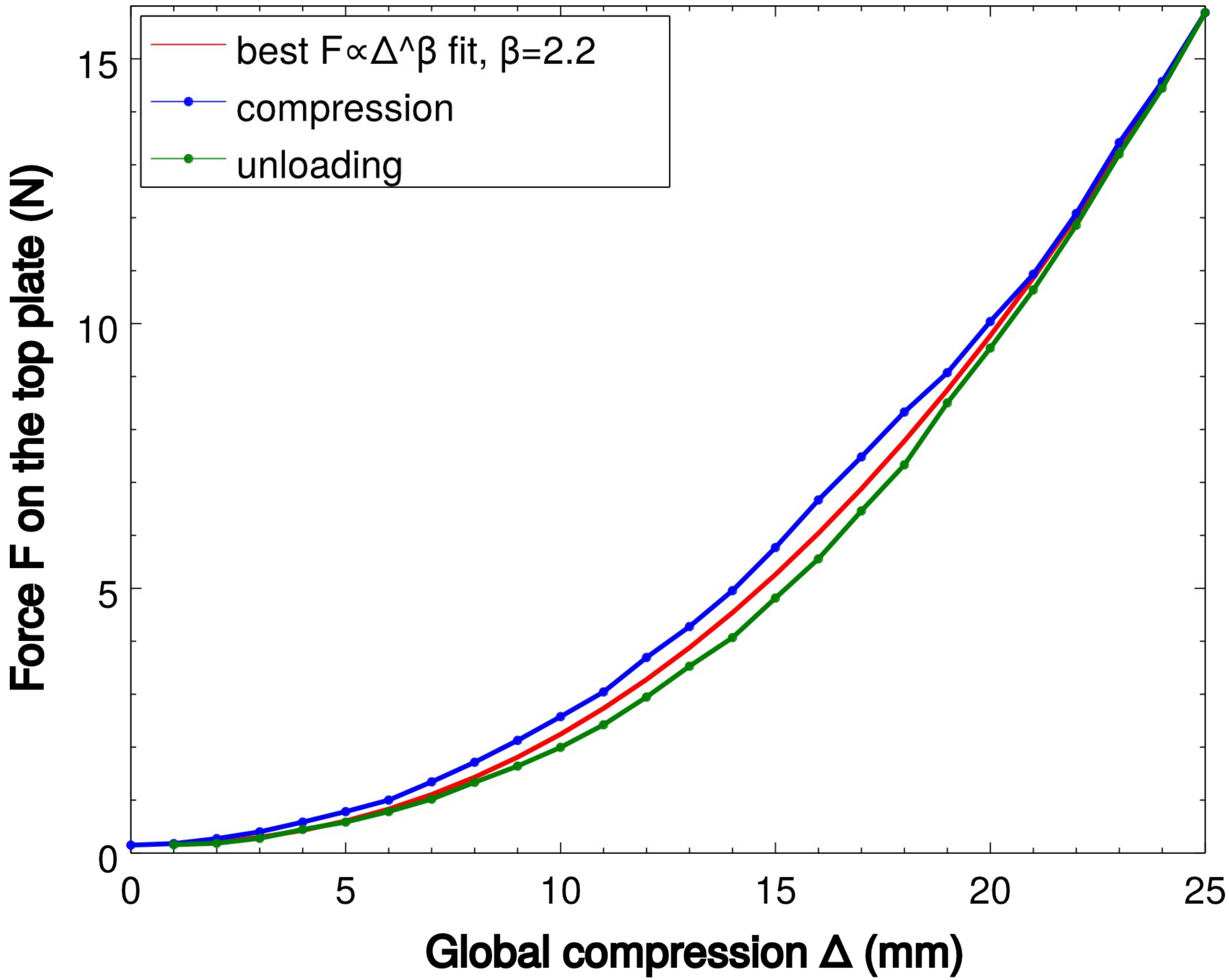
Green = force inferred from the images + measure of E≈23 kPa

Full experiment:

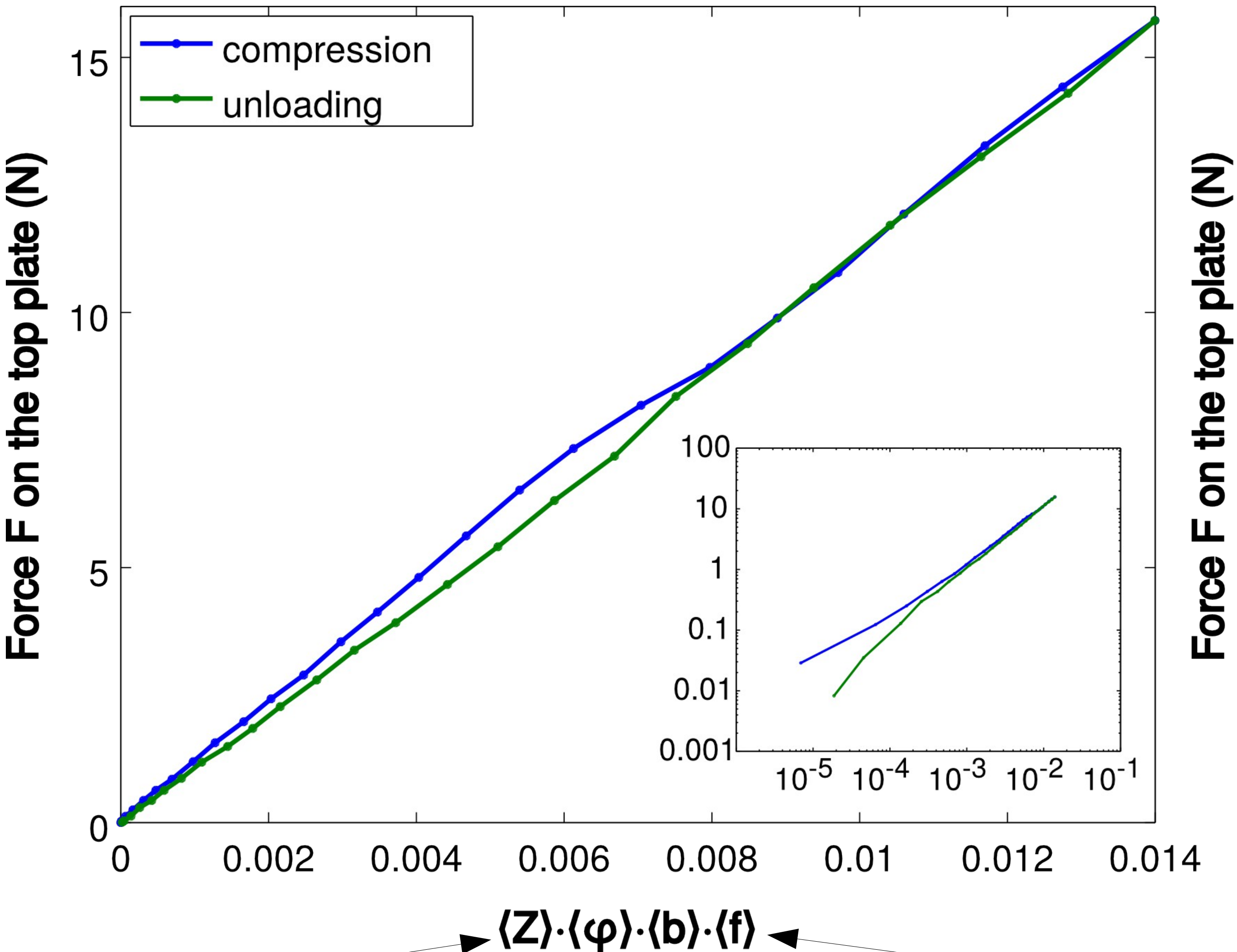
- Maximal compression of $\Delta \approx 13\%$ the initial height
- Average force at min compression $\approx 10^{-2}$ N
- 20 cycles = 1200 scans, >1M contacts (=good stats!).



Non-Hertzian Packing response



A SCALING HOLDS



$$F \propto \Delta^{\beta}$$

$$\updownarrow$$

$$f \propto \delta^{3/2}$$

Links Micro & Macro
$\Longrightarrow$ $\Delta(\delta)$ relation

Source of the scaling
= stress tensor
(details on demand)

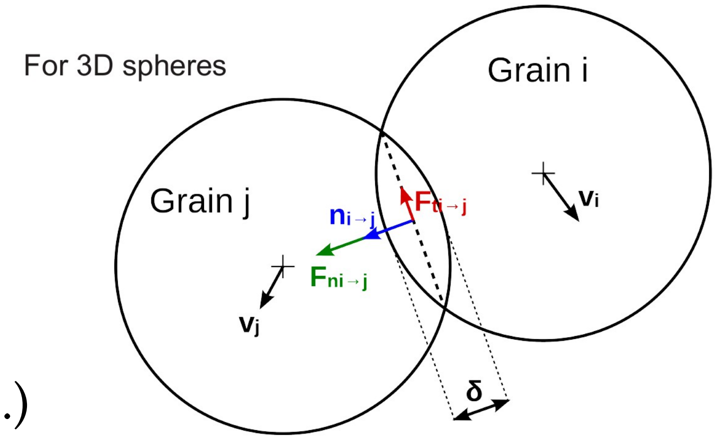
#contacts / grain

volume fraction

Distance between
grain centers

force at contacts

DEM of the "same" system



Standard DEM simulation

Overlapping non-deformable spheres, overlap = δ

+ Force model F = F(δ) (either linear, Hertz, hysteretic...)

+ Solid Dynamics (friction, solid bodies)

+ Newton's law for integrating the new positions after an elementary dt

= Trajectories of all particles + all contact properties

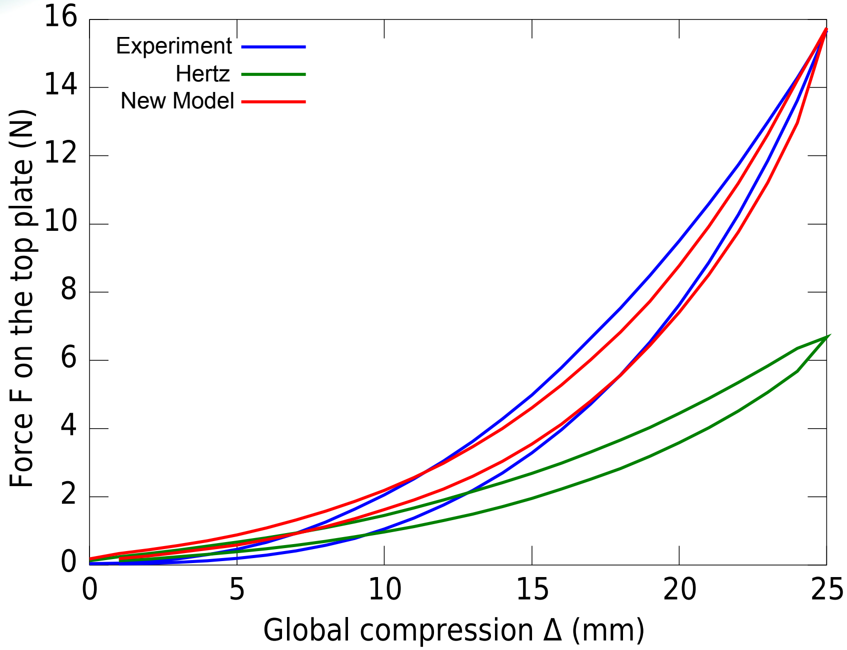
As close as we can get to the experimental setup

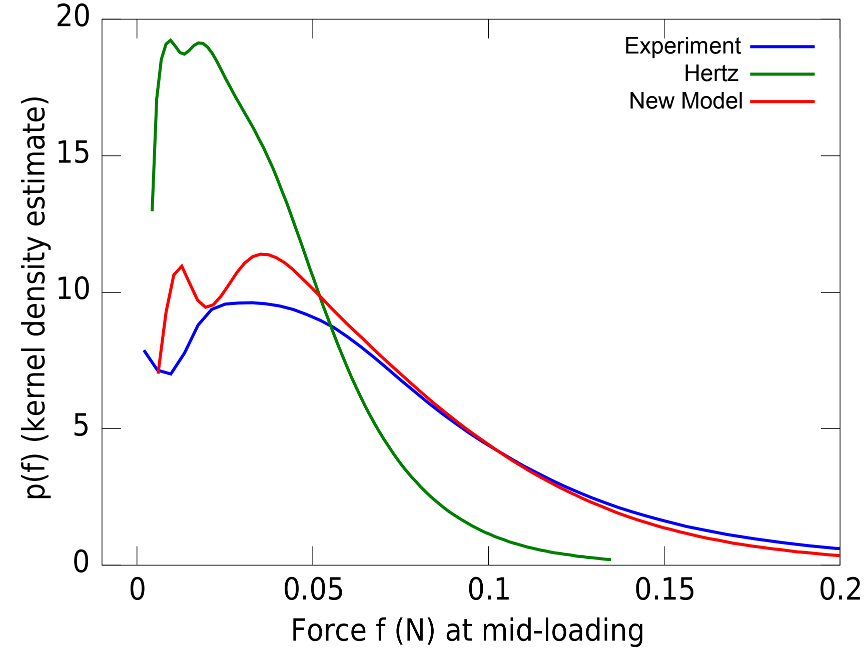
Hertz + same material properties

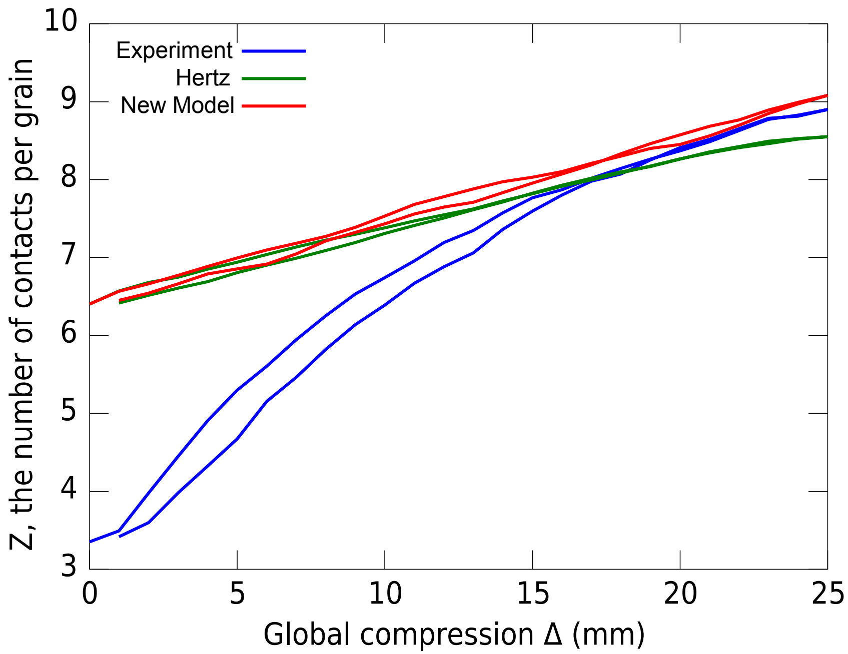
Grains are replaced by spheres of the same volume with the same center of mass

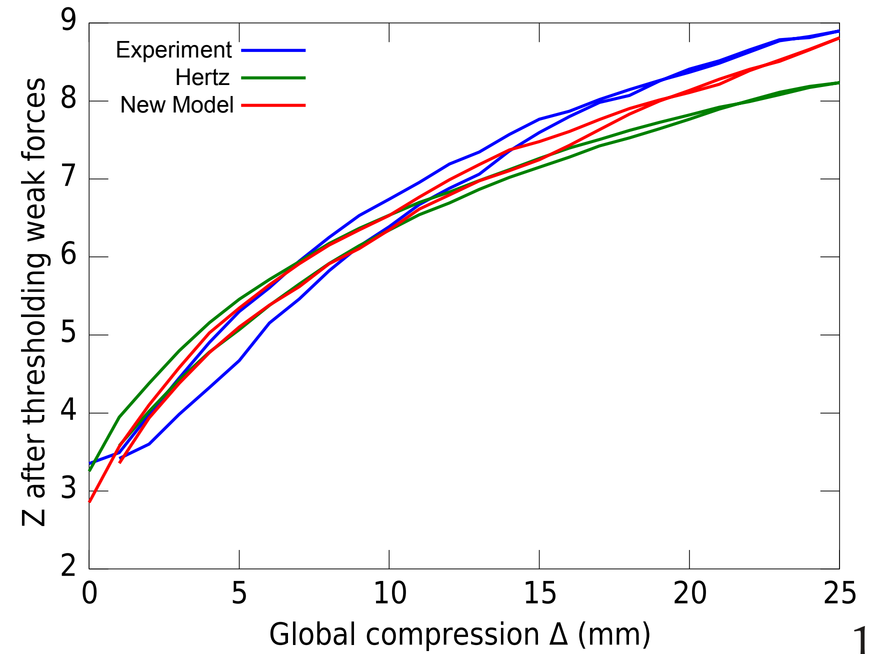
Same box size and piston motion

Comparison with data

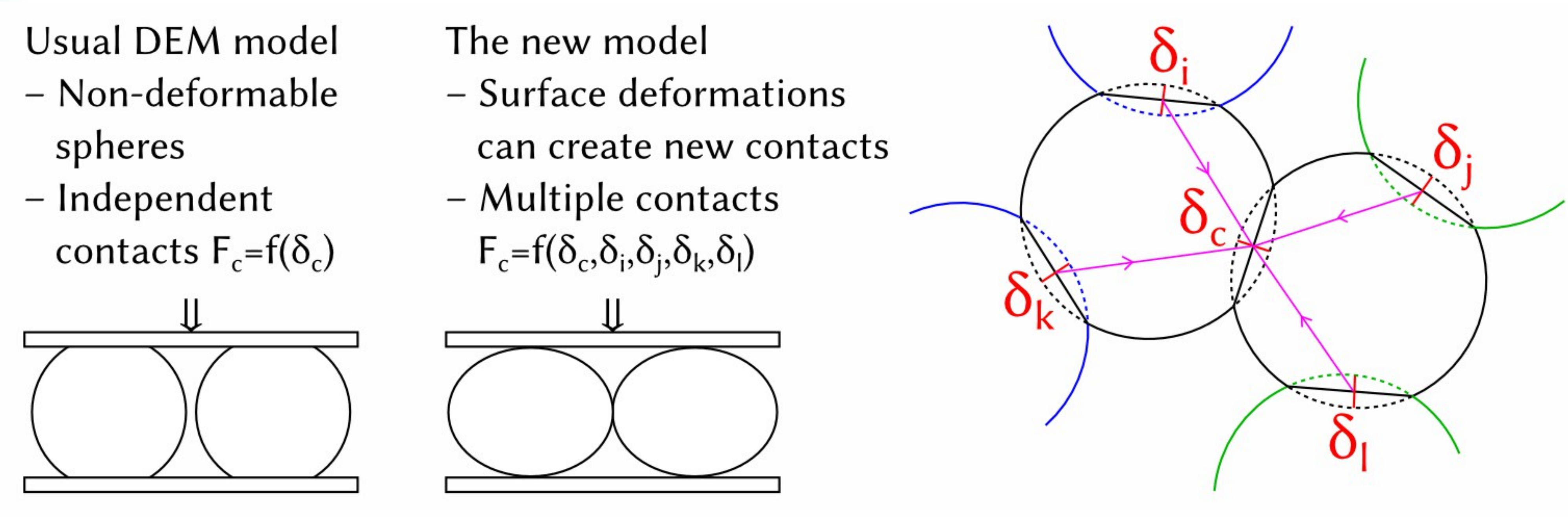








DEALING WITH MULTIPLE CONTACTS



Correlation between contacts (ex [Gonzalez/Cuitiño])

– δ_c = the deformation at contact c.

– Other contacts i,j,k,l have an extra influence on c

New model

– A different expression for the correlations

– Surfaces are deformed **before** the grains touch: new contacts can be created

– **Backed up by data!**

Induced displacements

Sphere-based ideas in the literature

- Point force on full sphere solution [Bondareva 1969]
- Gonzales / Cuitiño, approximating Zhapanska's solution for two spheres in contact

Problem:

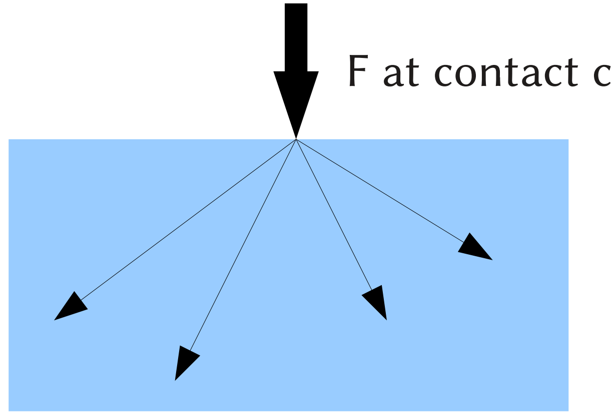
Dense granular material is more like an interconnected medium with pores

Very complex boundary (+inner) conditions.

DEM $\Longrightarrow$ Decoupling of the grains. Contact model = recoupling.

Here: Linear Elasticity on infinite half space

- Point force applied on elastic material
- Half-plane approximation = consistent with Hertz
- Geometric factor introduced in the model
  to compensate for the holes



Displacement δ induced by F at distance d
in the continuous elastic medium

The model

$$\delta_{k\to c} = -\frac{(1+\nu)\delta_k\sqrt{\delta_k r_k}}{2\pi d_{kc}}\left\{(\mathbf{n}_k\cdot\mathbf{u}_{kc})\,(\mathbf{n}_c\cdot\mathbf{u}_{kc}) + (3-4\nu)\mathbf{n}_k\cdot\mathbf{n}_c - (1-2\nu)\,\frac{(\mathbf{n}_k+\mathbf{u}_{kc})\cdot\mathbf{n}_c}{1+\mathbf{n}_k\cdot\mathbf{u}_{kc}}\right\}\times\gamma$$

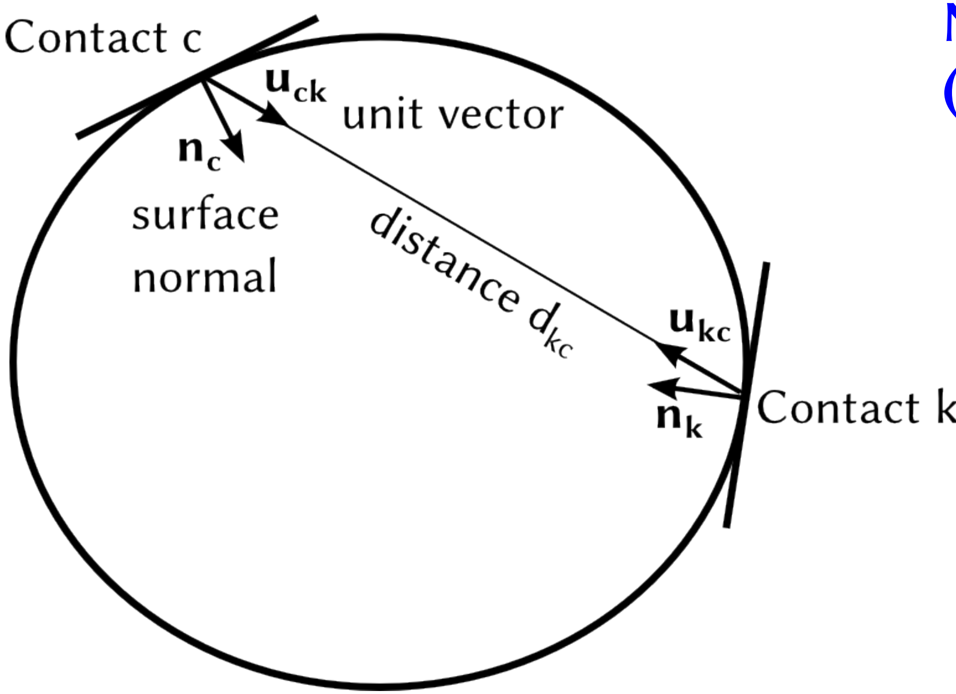
$$F_c = E^*\sqrt{r_c^*}\left(\delta_L + \sum_{k\neq c}\delta_{k\to c}\right)^{3/2}$$

Linear Elasticity

Geometric factor (to fit)

$\gamma \approx 1.19$

Non-local contributions (same as in [Gonzales/Cuitiño])



Notes

– $\delta_{k\to c}$ has no dependency on E

$\Rightarrow$ hard grains OK close to jamming

– γ may change with the packing fraction... not done yet

– Only validated for $\nu = 0.5$

QUESTIONS?